ENERGY LOSS IN PIPE FLOW

Energy losses in pipe flow

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of the fluid is lost.

Energy losses

Major energy loss
(due to friction)

Minor energy losses

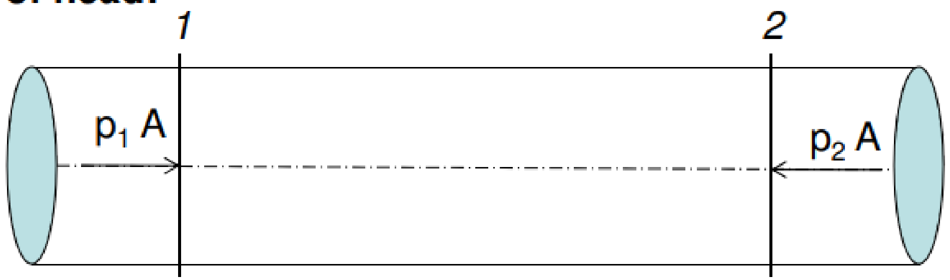
a. Sudden expansion
b. Sudden contraction
c. Bend in pipe
d. Pipe fitting
e. An obstruction in pipe

MAJOR ENERGY LOSS DUE TO THE FRICTION

Frictional losses in pipe flows

- The viscosity causes loss of energy in flows which is known as frictional loss.

Expression for loss of head:



Consider a horizontal pipe, having steady flow as shown above.

Let L = length of the pipe between sections 1 and 2.
d = diameter of the pipe
f = friction factor
h_f = loss of head due to friction.
p_1 = pressure at section 1
v_1 = *velocity* at section 1

p_2, v_2 are the corresponding values at section 2.

Applying Bernoulli's equations for real fluid at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{v_1^{\ 2}}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^{\ 2}}{2g}+z_2+h_f$$

But $z_1 = z_2$, and $V_1 = V_2$, as the pipe is horizontal and the diameter of the pipe is same in both sections.

$$h_f=\frac{p_1}{\rho g}-\frac{p_2}{\rho g} \qquad (1)$$

Darcy friction factor is defined as , $f=-\dfrac{\dfrac{dp}{dx}D_h}{\dfrac{1}{2}\rho V^2}$

Hydraulic diameter, $D_h=\dfrac{4(Area)}{Perimeter}$

$$\Rightarrow -\frac{dp}{dx}=\frac{f\rho V^2}{2D_h}$$

$$\Rightarrow p_1-p_2=\frac{f\rho V^2 L}{2D_h}$$

Using Eq. (1), we obtain

$$h_f = f \frac{L}{D_h} \frac{V^2}{2g}$$ **Darcy-Weishbach equation**

Loss of energy due to friction

Darcy-Weishbach equation: Head loss due to friction,

$$h_f = f \frac{L}{d} \frac{V^2}{2g} = 4C_f \frac{L}{d} \frac{V^2}{2g}$$

L: length of the pipe
V: mean velocity of the flow
d: diameter of the pipe

f is the friction factor for fully developed laminar flow:

$$f = \frac{64}{\mathrm{Re}} \; (for \;\mathrm{Re} < 2000) \qquad \mathrm{Re} = \frac{\rho u_{avg} d}{\mu}$$

C_f is the skin friction coefficient or Fanning's friction factor.

For Hagen-Poiseuille flow: $C_f = \tau_{wall} / \frac{1}{2} \rho u_{avg}^2 = \frac{16}{\mathrm{Re}}$

For turbulent flow:

$$\frac{1}{\sqrt{f}} = 1.74 - 2.0 \;\log_{10}\left[\frac{\varepsilon_p}{R} + \frac{18.7}{\mathrm{Re}\sqrt{f}}\right]$$ **Moody's Diagram**

R: radius of the pipe
ε_p : degree of roughness (for smooth pipe, $\varepsilon_p = 0$)
$\mathrm{Re} \to \infty$: completely rough pipe

MINOR LOSSES IN PIPES

Losses caused by fittings, bends, valves, etc

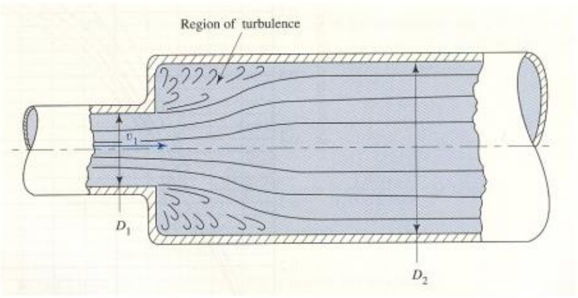
Minor in comparison to friction losses which are considered major.

Losses are proportional to – velocity of flow, geometry of device.

$$h_L = K(v^2 / 2g)$$

The value of K is typically provided for various devices. K - loss factor - has no units (dimensionless)

Sudden enlargement



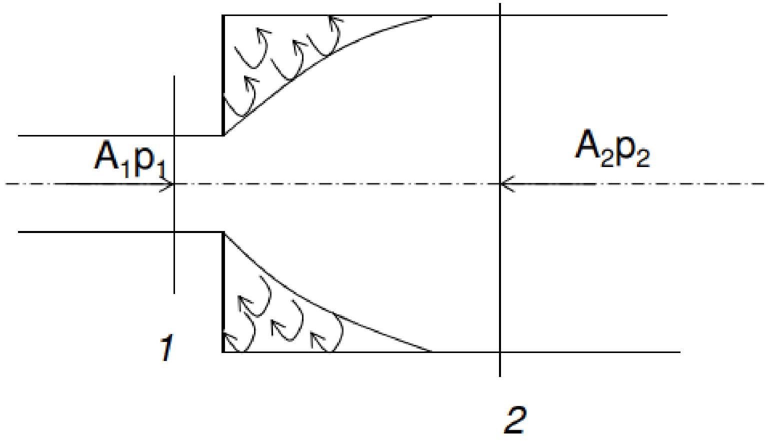
Energy lost is because of turbulence. Amount of turbulence depends on the differences in pipe diameters

$$h_L = K(v_1^2/2g)$$

If the **velocity v_1 < 1.2 m/s or 4 ft/s,** the K values can be given as

$$K = [1-(A_1/A_2)]^2 = [1-(D_1/D_2)^2]^2$$

Minor Energy head loss: Sudden expansion



Let A_1 = Area at section 1.
p_1 = pressure at section 1
v_1 = *velocity* at section 1

A_2, p_2, v_2 are the corresponding values at section 2.

Applying Bernoulli's equations for real fluid at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_e$$

But $z_1 = z_2$ as the pipe is horizontal.

$$\Rightarrow h_e = \frac{p_1 - p_2}{\rho g} + \frac{v_1^2 - v_2^2}{2g} \qquad (1)$$

MINOR LOSSES IN PIPES

Consider a control volume of liquid between sections 1 and 2. Resolving the forces acting on the liquid inside the control volume, we get

$$F_x = p_1 A_1 - p_2 A_2 + p'(A_2 - A_1)$$

where p' is pressure of the liquid eddies in the area (A_2-A_1). Experimentally it is known that $p' = p_1$, hence $F_x = (p_1 - p_2) A_2$

Momentum of liquid/sec at section 1= $\rho A_1 v_1^2$

Momentum of liquid/sec at section 2= $\rho A_2 v_2^2$

Change of momentum of liquid/sec = $\rho A_2 v_2^2 - \rho A_1 v_1^2 = \rho A_2 \left(v_2^2 - v_1 v_2\right) = F_x$

(Using the continuity equation)

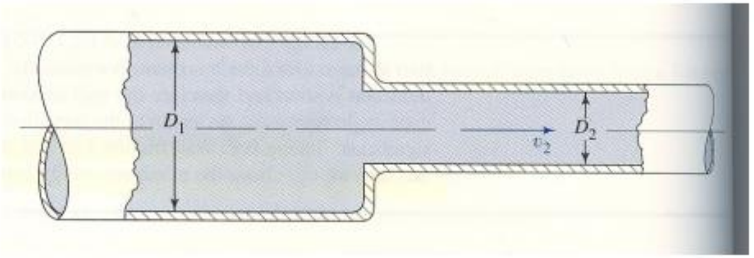
$$\frac{p_1 - p_2}{\rho g} = \frac{v_2^2 - v_1 v_2}{g}$$

Thus from (1), *we obtain*

$$h_e = \frac{(v_1 - v_2)^2}{2g}$$

Sudden Contraction

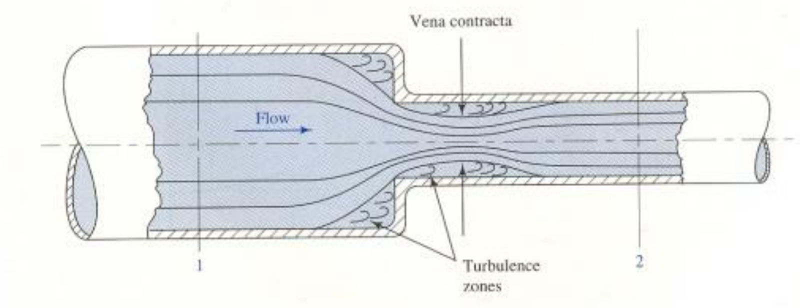
Decrease in pipe diameter –



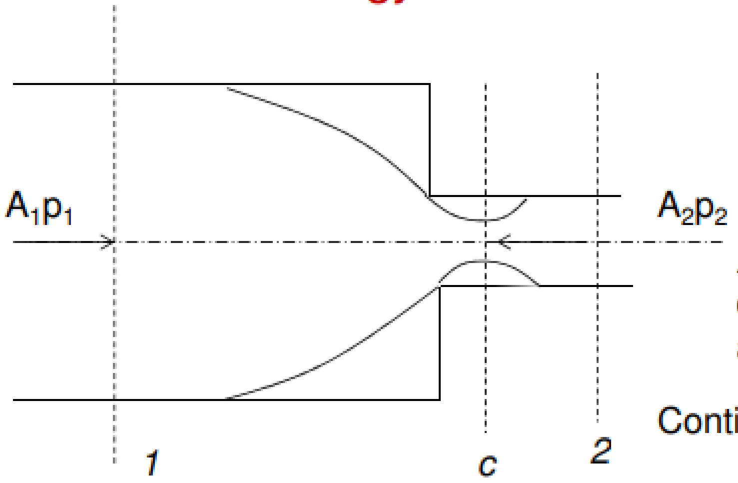
Loss is given by –

$$h_L = K(v_2^{\,2}/2g)$$

The **loss is associated with the contraction of flow and turbulence** –



Minor energy head loss: Sudden contraction



Let A_1 = Area at section 1.
p_1 = pressure at section 1
v_1 = *velocity* at section 1

A_2, p_2, v_2 and A_c, p_c, v_c are the corresponding values at section 2 and c, respectively.

Continuity equation:

$$A_c v_c = A_2 v_2 \Rightarrow \frac{v_c}{v_2} = \frac{A_2}{A_c} = \frac{1}{C_c}$$

Head loss due to expansion from section c to 2

$$h_c = \frac{(v_c - v_2)^2}{2g} = \frac{v_2^2}{2g}\left(\frac{v_c}{v_2} - 1\right)^2 = \frac{v_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2 = k\frac{v_2^2}{2g}$$

where, $k \equiv \left(\frac{1}{C_c} - 1\right)^2$. The value of k is 0.5 to 0.7.

Energy losses for sudden contraction are less than those for sudden enlargement

- **Head loss at the entrance of the pipe:** $h_i = 0.5\frac{v^2}{2g}$,

  where v is the velocity of the liquid in the pipe.

- **Head loss at the exit of the pipe:** $h_0 = \frac{v^2}{2g}$,

  where v is the velocity of the liquid at the outlet of the pipe.

- **Head loss due to bend in pipe:** $h_b = \frac{kv^2}{2g}$,

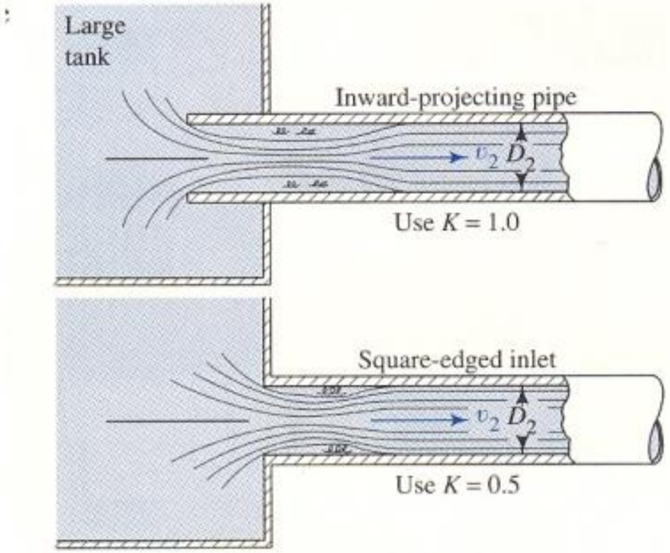
  where v is the velocity of the flow, k is the coefficient of the bend which depends on the angle of the bend, radius of curvature of the bend and diameter of pipe.

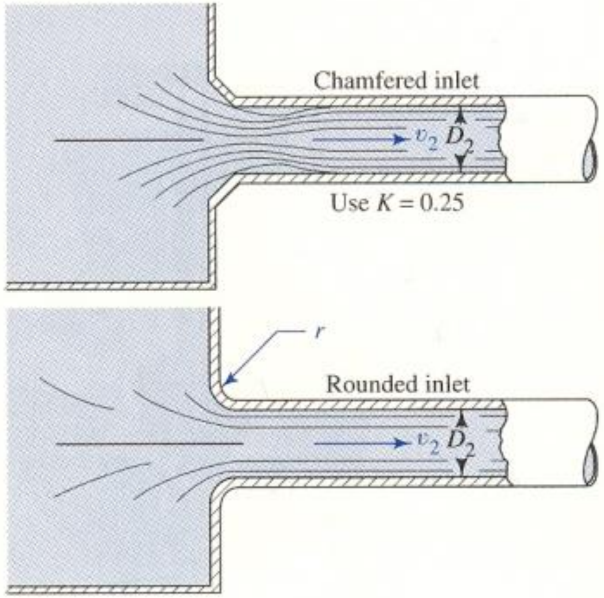
- **Head loss due to pipe fittings:** $h_f = \frac{kv^2}{2g}$,

  where v is the velocity of the flow, k is the coefficient of pipe fitting.

Entrance Losses

Fluid moves from zero velocity in tank to v_2





r/D_2	K
0	0.50
0.02	0.28
0.04	0.24
0.06	0.15
0.10	0.09
>0.15	0.04 (Well-rounded)

Resistance Coefficients for Valves & Fittings

Loss is given by –

$$h_L = K(v^2 / 2g)$$

Where K is computed as –

$$K = (L_e / D) * f_t$$

L_e = equivalent length (length of pipe with same resistance as the fitting/valve)

f_T = friction factor